

# De Noising of Images With Moment Invariants

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**Abstract---** In this paper we investigate a new technique which is used in Image Analysis, i.e., Moments. We specifically focus on the Hu's Seven Moments which are Rotationally Invariant. We have used this concept in the Non-Local Maximum Likelihood (NLML) Approach for de noising of Magnetic Resonance Imaging. In the existing NLML, a technique is used called Euclidean distance for finding intensity distances between pixels. But the major problem with this technique is that, it's not rotationally invariant. So, we have introduced the Moments concept by replacing the Euclidean Distance and have showed improvement in the efficiency of NLML approach, by undergoing various experiments with images and quantitative analysis on similarity measure with SSIM and PSNR.

**Keywords—** Structural Similarity Index Measure (SSIM), Peak Signal to Noise Ratio (PSNR), magnetic resonance (MR) images, Non Local Maximum Likelihood estimation (NLML).

## I. INTRODUCTION

Efforts in the direction of de noising of MR magnitude images is going on since a long time with several methods and approaches stated like Gaussian filters, Anisotropic Diffusion Filter (ADF), Modern wavelet based filters and local and non-local maximum likelihood estimation. The MR magnitude images are distorted by various noises by these sources: (i) thermal noise from the body (ii) quantization noise in the A/D devices (iii) electronic noise and (iv) thermal noise in the RF coil, which causes deterioration and uncertainties in the measurement of quantitative parameters, that hampers the estimation of the different properties of the analyzed tissues [1]. Hence to get a precise MR magnitude image is of utmost importance in medical diagnosis.

A new approach found recently called Non-Local Maximum Likelihood Approach has been the major area of interest in de noising field. Recently a better non-local maximum likelihood estimation algorithm approach came into existence which is called an Adaptive non-local maximum likelihood estimation Estimation [2]. The improvement is achieved by adaptively selecting a number of samples to be considered for the ML estimation. Through this approach, the over and under smoothing caused by the NLML can be reduced. Experiments have been carried out on simulated and real data sets. Quantitative analysis at various noise levels based on the similarity measures, PSNR, SSIM, BC and MAD shows that the proposed method is more effective than conventional NLML.

Experiments were also performed on real MR images to prove the efficacy of the proposed method [1].

As in whole in this approach, the samples for the ML estimation of the true underlying intensity are selected in a non local way based on the intensity similarity of the pixel neighborhoods i.e., there are likely to be numerous statistically identical yet disjoint regions distributed throughout the image. Based on this high degree of redundancy within an image, the assumption is made that pixels which have similar neighborhoods come from the same distribution. In other words, the intensity of pixel is predicted by using its NL neighborhood [1][3]. This similarity is generally measured using the Euclidean distance. But this Euclidean distance is a Non-Invariant Similarity Measure that means, if the pixels having same intensity are oriented in a different manner, then Euclidean would not consider them same and will eventually lead to an incorrect result. Thus, using Euclidean Distance as a similarity measure is not appropriate and should be replaced with a similarity measure which is Rotationally Invariant. We experimented with MISIM procedure i.e., Moment Invariant Similarity Measure and tried to implement the algorithm but it was not promising.

So we took the Hu's Moments and applied it solely in our further processing. After making rigorous experiments and calculations with various images, with different sizes, we came to a conclusion that Hu's Moments work quite well and applied them in non-local maximum likelihood estimation Approach and got more efficient results compared to the existing non-local maximum likelihood estimation. The noisy images using non-local maximum likelihood estimation having moments are more clearer than the noisy images of existing NLML. By taking Quantitative analysis on the similarity measure, SSIM. The **structural similarity** (SSIM) index is a method for measuring the similarity between two images. The SSIM index is a full reference metric; in other words, the measuring of image quality based on an initial uncompressed or distortion-free image as reference. The PSNR block computes the peak signal-to-noise ratio, between two images. This ratio is often used as a quality measurement between the original and a compressed image. The higher the PSNR, the better the quality of the compressed, or reconstructed image. We have made all experiments and coding in MATLAB with grayscale images, as MR magnitude images are in grayscale.

## II. HISTORY OF MOMENTS

Moments came into existence before the first computers, in the 19th century under the framework of the theory of algebraic invariants which was originated by famous German Mathematician David Hilbert[2].

Moment Invariants were firstly introduced to the pattern recognition community in 1962 by Hu. Since then Dudani, Wong, Flusser, Wallin, Teague, Suk, Reiss and others showed their perseverance in deriving independent sets of Invariants, derive invariants to general affine transform, etc., to gain an insight knowledge of the functioning of Moments[4][5][6]. With their efforts Moments were deployed in aircraft silhouette recognition, template matching and registration of satellite images, etc.[3][7][8].

## III. MOMENTS AS ROTATION INVARIANT

### 2.1 Basic Terms

First we define the basic terms :

Definition 1: By image function (or image) we understand any real function  $f(x, y)$  having a bounded support and a finite nonzero integral.

Definition 2: Geometric moment  $m_{pq}$  of image  $f(x, y)$ , where  $p, q$  are non-negative integers and  $(p + q)$  is called the order of the moment, is defined as

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx dy \quad (1)$$

Corresponding central moment  $\mu_{pq}$  and normalized moment and  $v_{pq}$  are defined as

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - x_0)^p (y - y_0)^q f(x, y) dx dy \quad (2)$$

$$v_{pq} = \mu_{pq} / \mu_{00}^w$$

respectively, where the coordinates  $(x_c, y_c)$  denote the centroid of  $f(x, y)$ , and  $\omega = (p + q + 2)/2$ .

As early as in 1962, M.K. Hu published seven rotation invariants, consisting of second and third order moments (we present first four of them):

$$\varphi_1 = \mu_{20} + \mu_{02} \quad (3)$$

$$\varphi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_2 \quad (4)$$

$$\varphi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2 \quad (5)$$

$$\varphi_4 = (\mu_{30} - \mu_{12})^2 + (\mu_{21} + \mu_{03})^2 \quad (6)$$

Major weakness of the Hu's theory is that it does not provide for a possibility of any generalization. By means of it, we could not derive invariants from higher order moments and invariants to more general transformations. These limitations were overcome thirty years later[9].

We have used orientation of images at angles 90, 180 and 270 degree because, Analysis of Hu's Moment Invariants on Image Rotation shows that the fluctuation of moments is minimum or becomes weak as the rotated angle is near to 90, 180, 270 and 360 degree[10].

### B. Non-local Maximum Likelihood Estimation Algorithm

Our NLML denoising scheme can be expressed as follows [13]:

NLML MR Denoising Scheme:

- **Input:** An MR image  $M_{sxt}$  and two odd numbers  $m$  and  $n$ ,  $n < m$ .
- **Output:** A denoised MR image  $\hat{A}_{sxt}$ .

- Identify a region from the background of the image, and estimate  $\sigma^2$  according to  $\hat{\sigma}_{ML} = \frac{1}{2n} \sum_{i=1}^n m_i^2$ .

- For each pixel  $x$ , take a window  $w_{m \times m}(x)$ .

- Within  $w_{m \times m}(x)$ , define a neighborhood  $n_{n \times n}(x)$ .

- For each  $y_i$ ,  $i = 1, 2, 3, \dots, n \times n$ , and  $y_i \neq x$ , compute

$$d_{xyi} \text{ according to } d_{xyi} = ||v(\eta_i) - v(\eta_j)||.$$

- Sort the list of  $D = \{d_{xyi}\}$  in increasing order.

- Form the NL neighborhood  $\zeta_x$  with  $y_{ind}$ , where  $ind$  is a list of the index of first  $k$  elements in  $D$ .

- Compute the log-likelihood function of the observations in  $\zeta_x$  according to

$\log L =$

$$\sum_{i=1}^n \log\left(\frac{m_i}{\sigma^2}\right) - \sum_{i=1}^n \log\left(\frac{m_i^2 + A^2}{2\sigma^2}\right) + \sum_{i=1}^n \log I_0\left(\frac{Am_i}{\sigma^2}\right)$$

- Estimate  $\hat{A}_x$  according to  $\hat{A}_{ML} = \arg\{\max_A(\log L)\}$ .

Here  $m$  is the sub image size,  $n$  is the size of a sliding window, We can set the size of  $m, n$ , and  $k$  to be 25, 3, or 5 and 25, respectively.

Now, we replace the Euclidean Distance used in computing the distance  $d$  with the invariant moments by applying Hu's moments.

### C. Procedure

Step 1. We will take different grayscale images (here we took 512x512 size images).

Step 2. Rotate it at angles 90, 180, and 270 degrees.

Step 3. Take out the Euclidean Distance between the original image and other images rotated at angles, mentioned in Step 2.

Step 4. Note down the results.

Step 5. Repeat Step 2 and 3 for different grayscale images.

Step 6. Apply Hu's Moments similarly as Euclidean Distance and repeat from Step 2 to 5.

Step 7. Note the results.

Step 8. Repeat from step 1 to 7 by taking (cropping) different section of images like 3 x 3, 5 x 5, 20 x 20, etc., and note the results.

Compare the results of Euclidean Distance and Moments result for rotational invariant measure conformity.

Step 9. As moments value did not change for any image as well as for any of their orientations, hence we conclude that Hu's Moments are a good rotationally invariant measure.

Step 10. Now we introduce noise (like Gaussian noise, Salt - Pepper, Shot, Quantization, noise respectively) with the previously taken images.

Step 11. Rotate it at angles 90, 180, and 270 degrees.

Step 12. Again apply the Hu's Moments and observe whether it proves its rotational invariant property or not.

Step 13. As it ascertains its property we prepare a function of Rotationally Invariant Moments, implementing Hu's concept.

Step 14. Replace this function with the Euclidean Distance used in existing NLML algorithm.

Step 15. Now we select an image and introduced noise in it.

Step 16. De noise it with existing NLML algorithm code as well as with the new Moments NLML code.

Step 17. Observe the de noised images from both the algorithms.

Step 18. Prepare a function for Structural Similarity and Index Measure (SSIM) and PSNR.

Step 19. Compare the two images with SSIM and PSNR, resulted from existing NLML and new Moments NLML code.

Step 20. Note the results in a table.

IV. EXPERIMENTS AND RESULTS



Fig 1. Images used in experiments.

Fig1. (3 images) shows the images which we have used in our experiments and Fig.2 ( i - iv) shows the noisy image we created with the different orientations .

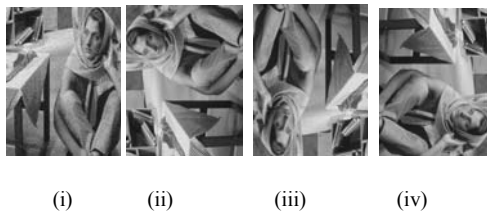


Fig 2. (i) Noisy Image, (ii) Image at 90 degrees, (iii) Orientation of 180 degrees, (iv) Orientation of 270 degrees.

TABLE I.

Comparison between Euclidean Distance and Moments with various images at different orientation (90°, 180°, 270°).

Image(A)	Image(B) Rotation	Euclidean Distance	Moment1 (A)
1.gif	None.	0	0.3883 0.0177
1.gif	90	153.4177	0.3883 0.0177
1.gif	180	162.6346	0.3883 0.0177
1.gif	270	152.8594	0.3883 0.0177
2.gif	None	0	0.3125 0.0977
2.gif	90	24.4540	0.3125 0.0977
2.gif	180	27.2029	0.3125 0.0977
2.gif	270	26.0768	0.3125 0.0977

TABLE II.

Quantitative analysis of PSNR and SSIM of images between Existing and new Moments NLML Method.

Image	SSIM (Existing NLML)	SSIM (Moments NLML)	PSNR (Existing NLML)	PSNR (Moments NLML)
1.gif	0.0710	0.1022	17.3389	17.7793
2.gif	0.0391	0.1098	20.9771	21.3959
10.gif	0.0394	0.1630	17.1103	19.2681

As mentioned in Step 17, we observe that the de noised image from New Moments NLML code is much clear then the existing NLML.

From Table1. it is quite clear that Hu's Moments, are a far more better Rotational Invariant Measure, as its value does not gets changed for any orientations of the images.

From Table2. we can observe that the SSIM and PSNR value is higher for the Moments based NLML Algorithm compared to the conventional NLML. Hence the higher the value the better is the quality of image.

V. CONCLUSION

By means of our experiments using the concept of Hu's Moments with different images and with their different orientations, eventually implementing it in the existing NLML Algorithm, a new horizon is opened for more better and precise pictures of the MR magnitude images which is vital for medical diagnosis in today's world. With the results from SSIM, PSNR and quality of pictures resulted from Moments NLML, we conclude that using moments improves the quality of MR magnitude images and also leaves further room to be researched with Hu's moments (which we have used here particularly) eventually leading to better results serving the society better. In the future, we will be doing further experiments with more similarity measures showing the clarity of images accurate results , and make the moment based Non Local Maximum Likelihood Estimation more efficient by improving its algorithm .

These experiments are performed with images just as the MR Images will be experimented with NLML Method. Hence next we will get the actual MR Images and eventually draw the concluded results.

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