ISSN:0975-9646

Barkuru Sri harsha et al, / (IJCSIT) International Journal of Computer Science and Information Technologies, Vol. 5 (3) , 2014, 3813-3816

B-Spline Non Rigid Image Registration using L-BFGS Optimizer

Barkuru Sri harsha ,PN Pournami , V K Govindan

Department of Computer Science and Engineering, National Institute of Technology Calicut, Kerala, India

***Abstract*—Image registration has many potential applications in clinical diagnosis like diagnosis of cardiac, retinal, pelvic, renal, abdomen, liver, and tissue diseases. It is a process of aligning images in order to monitor subtle changes. There are lots of image registration techniques evolved for soothing the image registration process. This paper proposes a B-spline non rigid image registration method using L-BFGS Optimizer.**

***Keywords*— Non-rigid image registration, B-spline transformation, Optimizer**.

I. INTRODUCTION

The major problem arises when the images taken from different viewpoints, by different sensors or at different times need to be compared. These images need to be aligned properly to identify differences. Similar problem occurs when we search for a small sub portion of image in another image. To find a good match, the proper alignment of the image with the sub portion must be found.

Image registration [1], [2], [3], [4] see is a process of aligning two images into a common coordinate system thus aligning them in order to monitor subtle changes between the two. Registration finds a transformation to set correspondence between the two images. Image processing methods are able to visualize part of the body, which are of special interest (this is, useful in medical treatments). In clinical diagnosis, information from different images is to be acquired for the correct diagnosis. For this, images need to be aligned properly for better observation. The process of mapping points from one image to respective points in another image is called Image Registration. In general, its applications can be divided into three main groups according to the manner of the image acquisition:

1. Different viewpoints (multi view analysis):

Images of the same scene are captured from different viewpoints. All the images acquired from different views are combined to form a 3D representation of the captured scene.

Applications: Computer vision shape recovery, mosaicing of images and Remote sensing.

2. Different times (multi temporal analysis):

Images of the same scene are captured at different times. This capturing is done on regular basis so that the changes in the scenes are identified.

Applications: Land-scape planning, Remote sensing monitoring of global land usage.

3. Different sensors (multimodal analysis):

Images of the same scene are captured by different sensors. As we are capturing information of the scene from different sensors we get more detailed and complex scene representation. Remote sensing is an important application of this category.

The general registration methods consist of the following four steps:

1. *Feature Detection:* Every image has distinct objects which can be used for comparisons like contours, corners, line intersections etc. Some of these distinct features of each image are detected.

2. *Feature Matching:* The correspondence between the features of the static image and moving image are established. In order to get this, we use different similarity measures.

3. *Transform Model Estimation:* The parameter of the function which transforms the moving image to the static image is estimated. This is mainly done by using feature comparison.

4. *Image Resampling and Transformation:* The moving image is transformed by mapping function obtained from Transform Model Estimation. Appropriate interpolation technique is used, if needed.

Most of the medical image applications are related with the Non-rigid image registration. Non-rigid image registration undergoes some series of space transformations to match the points of moving image with the corresponding points of static image. General medical image registration can be divided into three parts.

(1) First, to determine the space transformation of the source image and the target image;
(2) the second, for measuring the similarity degree of source image and target image, and
(3) the third to take some measures to make the similarity measure reaches the optimal value (parameter optimization) better and faster.

The rest of the paper is organized as follows: Section 2 introduces mutual information, one of the similarity criteria. Section 3 is about different transformations that we are going to use in this paper. Section 4 gives a brief introduction about the Line search optimization, emphasizing more on LM-BFGS. The proposed method is described in Section 5. Section 6 presents the results, and finally, Section 7 concludes the paper.

II. MUTUAL INFORMATION

A Mutual Information (MI) [5] is a basic concept from information theory, measuring the statistical dependence between two random variables or the amount of information that one variable contains about the other. Mutual information is huge when corresponding voxel pairs of the two images are geometrically aligned. Unlike the other similarity methods, MI does not consider the nature of relation between the intensities of the images to be registered and modalities [6] involved in capturing the images. So it is very powerful, widely used for multi-modal image registration technique.

I(A,B) = H(A) + H(B) – H(A,B)
I(A,B): The amount of information that B contains about A.
H(A) and H(B) being the entropy of A and B respectively.
H(A,B) is the joint entropy of A and B.

There are some cases where the MI values show very high even with large mismatch in the registration. This occurs when relative areas of objects and background overlaps, that is, the sum of the marginal entropies (H(A) + H(B)) increases faster than the joint entropy (H(A,B)). In order to avoid these anomalies, Studholmes [7] proposed a normalized measure of mutual information which is less sensitive to this overlaps.

$$C_{similarity}(A,B) = \frac{H(A)+H(B)}{H(A,B)}$$

III. TRANSFORMATION

In medical image registration, accuracy of image registration is very crucial. So, in this proposed method, we perform both Global and Local transformation [8].

$$T(x,y,z) = T_{global}(x,y,z) + T_{local}(x,y,z)$$

Global Transformation Model:

It describes the overall motion of the object. The simplest choice is a rigid transformation which is parameterized by 6 degrees of freedom, describing the rotations and translations of the object. Classes of general transformations are affine transformation, describing translation, rotation, scaling and shearing.

In 3-D, an affine transformation can be written as

$$T_{global}(x,y,z) = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{bmatrix}$$

Where, the coefficients parameterize the 12 degrees of freedom (Rotation, Translation, Shearing and Scaling).

Local Transformation Model:

As the Global transformation model can't deal with deformation of the image to be registered, we go for this transformation model.

To correct the deformation in the image we go for Free Form Deformation (FFD) model using B-spline [9] which is a powerful tool for modeling 3-D deformable objects. The basic idea of FFD is to deform an object by manipulating an underlying mesh of control points.

In Spline based FFD, we denote the domain of the image volume as Ω= {(x,y,z)| 0≤x<X,0≤y<Y,0<z≤Z}. Let $\emptyset_{i,j,k}$ with uniform spacing $\boldsymbol{\delta}$. The FFD can be written as the 3-D tensor product of the familiar 1-D cubic B-splines.

$$T_{local}(x,y,z) = \sum_{l=0}^{3}\sum_{m=0}^{3}\sum_{n=0}^{3} B_l(u)B_m(v)B_n(w)\emptyset_{i+l,j+m,k+n}$$

Where i=floor($\frac{x}{n_x}$) -1, j=floor($\frac{y}{n_y}$)-1, k=floor($\frac{z}{n_z}$) -1.

u = $\frac{x}{n_x}$-floor($\frac{x}{n_x}$), v = $\frac{y}{n_y}$-floor($\frac{y}{n_y}$) ,
w= $\frac{z}{n_z}$-floor($\frac{z}{n_z}$)

Where, B_l represents the l^{th} basis function of the B-spline. $n_x \times n_y \times n_z$ is the dimension of the control point grid.

$$B_0(u) = (1-u)^3/6$$
$$B_1(u) = (3u^3 - 6u^2 + 4)/6$$
$$B_2(u) = (-3u^3 + 3u^2 + 3u + 1)/6$$
$$B_3(u) = u^3/6$$

There are many spline based methods like thin-plate splines and elastic-body splines. These methods are globally controlled, change in control point $\emptyset_{i,j,k}$ at one position in turn effects the position of the most control points. When comes to B-spline, they are locally controlled. Change in control point effects the transformation only in the local region of the control point. B-spline is computationally efficient even for a large number of control points.

The control points ∅ act as parameters of the B-spline Free Form Deformation. Degree of non-rigid deformation can be tuned by changing the resolution of the mesh of control points ∅. With increasing spacing of control points, transformation tends to modeling of global non-rigid deformation. Small spacing of control points allows modeling of global non-rigid deformation. Factors like degrees of freedom, computational complexity depends on the resolution of the control point mesh. For spline-based FFD transformation to be smooth, penalty term is always added to the cost function. The penalty term is described by wahba[10].

IV. LINE SEARCH OPTIMIZATION

The line search approach first finds the direction of the objective function f along which the function is reduced. After that, it computes the step size to determine how far it should move along the given direction. There are numerous methods for computing the descent direction like Quasi-Newton method, Gradient descent and Newton's method.

Quasi-Newton methods are used to find maxima and minima of objective functions. As this method is based on Newton methods, we find the stationary point of the function. Generally Newton's method uses a matrix of second derivatives and the gradient of the function f.

Limited-memory BFGS [11] (L-BFGS or LM-BFGS) is an optimization algorithm in the family of Quasi-Newton methods that approximates the Broyden Flecther Goldfarb Shanno (BFGS) algorithm using a limited amount of computer memory. Conventional BFGS stores 'n' approximation of inverse hessian, where n denotes the number of variables in the problem. L-BFGS stores only few vectors, from which we can approximate the current values. As we are storing only few vectors, it results in linear memory requirement. So, the L-BFGS method is particularly suited for problems with large number of variables.

Basic optimization algorithm

Input: Initial x_0 value, f (function), k=0,
Stopping criterion: $\frac{df}{dx} = 0$
Output: x_k(is the value at which the function is a minimum)

Algorithm:

1. Compute a search direction p_k.
2. Compute the step length α_k, such that $f(x_k + \alpha_k p_k) < f(x_k)$.
3. Update $x_k = x_k + \alpha_k p_k$
4. Check for convergence (stopping criteria)

The above mentioned algorithm is the more generalized optimization algorithm.

BFGS algorithm is an iterative method for solving unconstrained nonlinear optimization problems. It approximates Newton's method, a class of hill-climbing optimization technique that seeks a stationary point of a function. A necessary condition for optimality is that the gradient must be zero.

Broyden-Flecther-Goldfarb-Shanno (BFGS)

Input: Initial value - x_0, δ - error, c_0- intial Hessian.
Stopping criterion: norm of gradient greater than zero.
Output: x_j (the value at which function is a minimum)

Algorithm:

1. j←0
2. while (true) do
   a. $d_j \leftarrow -c_j \Delta f(x_j)$
   b. $\alpha_j \leftarrow$ LineSearch(x_j,f)
   c. $x_{j+1} \leftarrow x_j + \alpha_j d_j$
   d. Compute c_{j+1}
   e. j←j+1;
   f. if $||\Delta f(x_j)|| \leq \delta$ Then
      stop
   g. end if
3. end while

BFGS is the most popular and effective Quasi-Newton update formula. Whole credit of the BFGS algorithm depends on the updating formula for C_j. C_j approximates the inverse f (the true Hessian at the current iteration). It has a self-correcting property (At some iterations, the matrix contains bad curvature information, it often takes only a few updates to correct these inaccuracies, provided a correct line search is used), which is very important for any optimization algorithm. BFGS requires only matrix-vector multiplications which brings the computational cost at each iteration from O(n^3) for Newton's method down to O(n^2). However, if the number of variables is very large, even O(n^2) per iteration is too expensive - both in terms of CPU time and sometimes also in terms of memory usage (a large matrix must be kept in memory at all times).

Limited-Memory BFGS [12] is computationally less intensive, when n is large. Generally entire Hessian c_j matrix is updated and stored, but LBFGS method never do that instead it store the information from the past 'm' iterations and uses that to compute the inverse Hessian (particularly for finding the next search direction). The updating in LBFGS is done using just *4mn* mulitplications, which brings the time complexity to O(mn) per iteration. If 'm' is much less than 'n', time complexity goes down to O(n). In some cases the LBFGS method uses as many or even fewer function evaluations to find the minimizer. Even using same number of function evaluations, LBFGS runs significantly faster than full BFGS if n is large.

Direction finding in LBFGS

q - The present gradient
y - The gradient vector, which contains all the previous gradient values.
v - The list of x values calculated in the process of getting optimum.
r = $\frac{Transpose\ of\ y * Transpose\ of\ v}{\sum y\text{^}2}$
s = Transpose of v.

Input: q (gradient), p = $\frac{1}{(Tranpose\ of\ y) * v}$, s = Transpose of v, y ,r.

Output: Direction d (d← $-r$)

Algorithm:

1. for i=(j-1): -1 : (j-m) do
2. $\alpha_i \leftarrow p_i (s^i)^T q$
3. q ← q-$\alpha_i y^{(i)}$
4. end for
5. for i= (j –m) : 1 : (j-1) do
6. $\beta \leftarrow p_i (y^i)^T r$
7. r← $r + (s^i)(\alpha_i - \beta)$
8. end for

V. PROPOSED METHOD

B-spline is more flexible transformation having high degrees of freedom. L-BFGS is an optimization algorithm which calculates it's parameters from the previous instances with linear memory storage.

By combining these two, we can get an efficient registration technique.

Algorithm:
Input: Target image (I1) and Reference image (I2)
Output: Registered image (I3)

1. Take the initial transformation matrix θ_0.
2. Compute 'θ_1' by using Global transformation.
   i.e $\theta_1 = \text{GlobalTransformation}(\theta_0)$
3. Initialize the control points $\emptyset$.
4. for i = $\emptyset^l$ to $\emptyset^n$
   a. Compute 'θ_2' by using local transformation.
      i.e θ_2=LM-BFGS(f, θ_1, opt)
   b. Increase the control point resolution by calculating the new control points
5. Apply B-spline transformation on the target image using θ_2 to obtain I3.

We can use any one of the global transformations like rigid, affine. Opt is a structure that contains information like error (δ), termination tolerance of transformation matrix. Values of 'l' and 'n' should be powers of 2. We fix the value of n. Function f contains the parameters like size of control points, I1 and I2.

VI. RESULTS

Figure1 & Figure 2 are Moving and Static image respectively. Figure 3 is the final registered image.

Table 1 shows the comparative results of the proposed method with affine transformation. The results clearly indicate the accuracy of the proposed approach over the affine method.

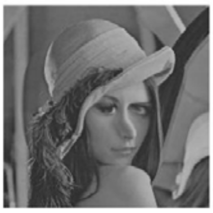

Figure 1: Moving image

Figure 2: Static image

Figure 3: Registered image

Table1: Compared results of Affine with proposed method.

	Image difference	RMS
Affine	20.08	9.92
B-spline with L-BFGS	1.16	2.27

VII. CONCLUSION

This paper presented a registration approach using B-spline transformation on the image so that any deformations present in the image can be rectified. In order to perform the registration process efficiently using B-spline, an optimization process is employed for achieving high accuracy and efficiency of registration. L-BFGS optimization technique is utilized here. The results obtained demonstrate that the proposed method is accurate and efficient.

REFERENCES

[1] L.G. Brown, "A survey of image registration technique" ACM computing surveys(CSUR), Vol.24, no. 4, pp. 325-376, 1992.

[2] Sithara Kanakaraj, Govindan V K and Pournami P N, A fast brain image registration using axial transform; Communications in Computer and Information science, 2012, vol. 292, part 2, pp. 206-212, DOI: 10.1007/978-3-642-31686-9_24.

[3] Pournami P.N, Sithara Kanakaraj & V.K. Govindan; Registration using multi-resolution optimization for the early diagnosis of alzheimer's disease; International Journal of Computer Science and Electrical Engineering (IJCSEE) ISSN No. 2315-4209, Vol-1, Iss-2, 2012.

[4] Pournami P N, Shaj G S, and V K Govindan, Threshold Accepting Approach for Image Registration, International Journal of Computer Science and its Applications, Vol.2, Issue 2, pp 88-92, 2012, ISSN 2250-3765.

[5] R.W.So and A.C.Chung, "multi-modal non-rigid image registration based on similarity and dissimilarity with the prior joint intensity distributions" in Biomedical Imaging: From Nano to macro, 2010 IEEE International symposium on pp.368-371, IEEE, 2010.

[6] E. Ardizzone, O. Gambino, M. La Cascia, L. Lo presti, and R.Pirrone, "Multi-modal non rigid registration of medical images based on mutual information maximization" in Image Analysis and processing, 2007. ICIAP 2007. 14th international Conference on, pp. 743-750, IEEE, 2007.

[7] C. Studholme, D. L. G. Hill, and D. J. Hawkes, "An overlap invariant entropy measure of 3D medical image alignment" *Pattern Recognition*, vol. 32, no. 1, pp. 71–86, 1999.

[8] D.Rueckert, L.I. Sonoda, C.Hayes, D.L.Hill, M.O. Leach, and D.J. Hawkes, "Nonrigid registration using free-form deformations: application to breast MR images" Medical imaging, IEEE Transactions on, vol.18,no. 8, pp.712-721,1999

[9] S.Lee, G. Wolberg, and S.Y.Shin, "Scattered data interpolation with multilevel b-splines" visualization and Computer Graphics, IEEE Transaction on, vol. 3, no. 3,pp. 228-244,1997

[10] G.Wahba, Spline models for observational data, vol.59.siam, 1990

[11] J.L.Morales, "A numerical study of limited memory bfgs methods" Applied Matheatics Letters, vol. 15, no.4, pp. 481-487,2002.

[12] http://cs.nyu.edu/overton/mstheses/skajaa/msthesis.pdf