

A Review on Transform Domain Adaptive Filters

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Abstract— *In this paper, transform domain adaptive filters are studied and reviewed based on previous researches. In transform domain adaptive filters orthogonality properties of transformation such as discrete cosine transform (DCT), discrete sine transform (DST), wavelet transform and discrete fourier transform (DFT) are used to achieve an improved convergence rate as compared to the time domain analysis. Also, it provides better computational speed, fast convolution as compared to time domain algorithms. TDAF are applied when long memory or long duration impulse response is required since it increases the computational complexity in time domain. A tabular format of review is also given for all transform domain adaptive filters algorithms, characteristics and their application areas.*

Keywords— **Tranform doimain adaptive filter, convergence rate, introduction, discrete cosine transform, discrete sine transform, Fourier Transform.**

I. INTRODUCTION

Adaptive filter is frequently used for many applications, like feedback cancellation, linear predictions, active noise control (ANC) and acoustic echo cancellation (AEC). There are different algorithms used for the realization of adaptive filters such as least mean square (LMS), recursive least square (RLS) and their variants. But, LMS and its variants are most popular due to their computational simplicity and stability. LMS was proposed by Hoff and Windrow. Therefore, LMS algorithm is used in various applications of adaptive filter such as, acoustic echo cancellation (AEC). But, it is not considered useful when a long echo duration is present as in the case of massive teleconferencing. In massive teleconferencing [1], long impulse response or long memory is required to cope up with the long duration of echo. LMS algorithm in time domain do not have long memory to cope up with the long duration echo therefore it causes problem of increased computational complexity. To resolve this problem infinite duration impulse response (IIR) in time domain [2], [3] can be chosen but in this a new difficulty arose, that is the stability problem. Therefore, to resolve all these problems transform domain adaptive filter (TDAF) was introduced. It provides better computational speed, fast convolution, enhances the convergence performance of the time domain LMS algorithm. TDAF uses the orthogonality properties of discrete cosine transform (DCT), discrete sine transform (DST), wavelet transform (WT) and discrete fourier transform (DFT) to achieve a much more improved convergence rate [4].

Fourier transform converts the time domain signals in to frequency domain signals. The frequency domain signal is converted back to time domain signal by the help of inverse

fourier transform. Here, the filter parameter alteration become more feasible in comparison to the time domain LMS algorithm. This concept was found in the research paper of Walzman and Schwartz in 1973. There are two methods widely used for frequency domain adaptive filtering [5], [6]. Firstly the block implementation of an FIR filter, in this parallel processing is used and due to this computational speed increases. Second, the Fast fourier transform (FFT) algorithm introduces fast convolution, the filter parameters are adapted in frequency domain in an efficient manner.

II. REVIEW ON TRANSFORM DOMAIN ADAPTIVE FILTER

B.Farhang-Boroujeny and S.Gazor [7] in January 1992, studied the quantization effects of transform domain normalized LMS (TDNLMS) algorithm. They have reviewed the fact that quantization has low sensitivity level in case of transformation coefficients. By performing this analysis, effective and robust implementation was achieved. The system implemented performs orthogonal transform for converting its input samples in to partially uncorrelated sets. Alberto Carini and Enzo Mumolo [8] in August 1999, proposed the use of unit diagonal (UD) in factorization in recursive least square (RLS) algorithm because it is numerically stable, its mean and mean square variance are smaller than square-root RLS algorithm. The fast RLS algorithm is free from square root. This algorithm is used for analyzing many data signals. Self-orthogonalizing transform domain least mean square (SO-TRLMS) algorithm was analyzed by Chong-ni Li, Guang-rui Hu and Min-jie Liu [9] in March 2000. SO-TRLMS provided good convergence speed in comparison to the transform domain least mean square (TRLMS) algorithm. This technique makes transform domain adaptive filter reliable for real time applications. SO-TRLMS is computationally simpler than time domain LMS. Attallah and S. W. Liaw [10] in June 2001, presented a new approach of discrete cosine transform LMS algorithm (DCT-LMS). Full update DCTLMS (FU-DCTLMS) and partial update DCTLMS (PU-DCTLMS) algorithms were analyzed. PU-DCTLMS reduced the computational complexity of conventional LMS algorithms. It shows excellent results for real signals and markov first order.

Kheong Sann Chan and Berhouz Farhang-Boroujeny [11] in September 2001, analyzed the partitioned frequency domain block least square algorithm (PFBLMS) on a new platform of matrices. These matrices are used to control the convergence rate. It evaluates the eigenvalue for both colored and white input signal. There are various matrices used in the implementation of frequency domain block

LMS (FBLMS) that are: normalized constrained FBLMS algorithm and normalized unconstrained FBLMS algorithms. If two matrices will have same eigenvalues then they will be asymptotically equivalent. But, unconstrained PFBLMS have slow convergence rate. To remove this, error scheduling technique was applied to various partitions due to this computational complexity decrease but convergence rate almost remain same as in case of constrained PFBLMS algorithm. Another type of constrained PFBLMS that is referred is schedule constrained PFBLMS algorithm.

Analysis of fast predictor based least square (FPLS) algorithm was carried out by Kazushi Ikeda, Shigemitsu Tanaka and Youhua Wang [12] in January 2002. By performing the analyzes it was proved that FPLS has slow convergence performance in comparison to the recursive least mean square (RLS) algorithm, even if the RLS input signal satisfies autoregressive assumption. Fast newton transversal filter (FNMF) algorithm is most reliable for implementation as it extends its range from LMS algorithm to RLS algorithm.

Transform domain LMS algorithm with variable step size (TDVSS) was proposed in February 2002 by Radu Ciprian Bilcu, Pauli Kuosmanen and Karen Egiazarian [13]. The speed of convergence increases in TDVSS in comparison to the standard TDLMS. Yiteng (Arden) Huang and Jacob Benesty [14] in January 2013, extended their studies based on time domain blind channel identification to frequency domain. Multichannel frequency domain LMS (MCFLMS) and normalized multichannel frequency domain LMS (NMCFLMS) algorithm were proposed. Convolution and correlation operation where performed in time domain multichannel LMS (MCLMS) are computationally intensive but by using overlap-save method and fast fourier transform (FFT) in frequency domain, MCLMS and multichannel newton (MCN) methods can be efficiently implemented and hence MCFLMS is derived. They have proved that the frequency domain approach is much better than time domain. NMCFLMS is used in signal processing. Partitioned block frequency domain adaptive filter (PBFDAF) analysis is performed by Koen Eneman and Marc Moonen [15] in March 2003. A fast version of row action projection and PBFDAF were combined to form a new version named as PBFDAF. It provides better system estimation than conventional PBFDAF. Alias free Sub-band adaptive filter (SADF's) was firstly given by Pradhan and Reddy. Shigeyuki Miyagi and Hideaki Sakai [16] in January 2004, analyzed the SADF algorithm in frequency domain. The analysis was done by the help of discrete fourier transform ordinary differential equation (ODE) and averaging method. This methodology was firstly applied to pradhan's SADF algorithm which proves that SADF is mostly stable and EMSE is smaller than the full-band adaptive filter. Initial technology was modified and two-band delayless subband adaptive filter (DLSADF) with hadamard transform was applied. Slow convergence rate was obtained in some of the cases.

K. Mayyas and T. Aboulnasr [17] in March 2004, proposed a new transform domain (TD) with low

complexity. In this approach they have used selective coefficients update (SCU) approach to reduce the computational complexity. The long length adaptive filter is divided in to small sub filters for making it convenient to use in acoustic echo cancellation (AEC). As TD has fast convergence and SCU has low computational complexity therefore SCU and TD are combined in this new approach to decrease the performance losses. As the convergence speed increases miss-adjustment also increases. To remove this problem hybrid algorithm was implemented. Hybrid algorithm has fast convergence speed and better performance than standard TDLMS algorithm. It provides less computational complexity. The LMS algorithm and its variants have high computational complexity if incase its filter length is large. Fourier transform based block normalized LMS (FBNLMS) was introduced to reduce the computational complexity by using discrete fourier transform (DFT). But, FBNLMS still have high computational complexity therefore, Hartley transform based normalized LMS (HBNLMS) was implemented by Vasanthan Raghavan, K. M. M. Prabhu and Piet C. W. Sommen [18] in February 2005, by using cosine (DCT) and sine (DST) symmetric decomposition of discrete Hartley transform (DHT) that reduces the FBNLMS computational complexity by 33%. A new Fast block-exact LMS (FELMS) was proposed by Y. Zhou, S. C. Chan and K. L. Ho [19] in January 2006. It is performed by using the LMS/Newton algorithm whitened input and then applying shifting property. New block-exact fast LMS update is carried out in the same way as that of LMS. This proposed algorithm has less computational complexity but they are equivalent to fast LMS in terms of mathematical stability. FWTDLMS with partial sub-band coefficients update (FWTDLMS-PU) was proposed by Samir Attallah [20] in January 2006. Fast wavelet transform (WT) domain LMS (FWTDLMS) algorithm is exercised to make the proposed algorithm. Elen Macedo Lobato, Orlando José Tobias and Rui Seara [21] in May 2008, proposed a concept of stochastic modeling. This concept was applied to TDLMS algorithm. The proposed algorithm is independent of filter order and type of orthogonal transform. Shengkui Zhao, Zhihong Man, Suiyang Khoo and Hong Ren Wu [22] in January 2009, stated a new approach of applying second order autoregressive (AR) process on transform domain least mean square (LMS) adaptive filters. By applying Power normalization and data independent orthogonal transform, convergence rate of adaptive filter is ameliorated.

Chandrasekhar Radhakrishnan and William Kenneth Jenkins [23] in January 2010, demonstrated that the fast-fourier transform (FFT) based fault-tolerance adaptive filters (FTAF). It was shown that same degree of fault tolerance without considering zero padding redundancy. Yekutiel Avargel and Israel Cohen [24] in October 2009, proposed short-time fourier transform domain (STFT) algorithm for non-linear system identification. When the power ratio from non-linear to linear become high then the estimation of mean square error (MSE) is improved by non-linear components. Cross band filter are present between sub-band. The steady state MSE have slower convergence rate when there is an increase in cross-band filters. To

reduce the problem of stereo acoustic echo cancellation (SAEC) Sheng Wu, Xiaojun Qiu and Ming Wu [25] in March 2011, proposed a windowing frequency domain adaptive filter (WDAF) and up-sampling block transform processor. Windowing filter has a property of smooth cut off to decrease the spectral leakage at the time of filter update. Therefore, the amount of independent noise in stereo acoustic echo cancellation introduced by pre-processing, can be increased. Windowing adaptive filter performs better than that of conventional FDAF in both cases whether it is stereo or mono.

Sarmad Malik and Gerald Enzner [26] in September 2012, presented a new approach for acoustic echo cancellation (AEC). This new approach was implemented in the presence of unknown memory less non-linearity preceding its echo path. By taking the non-linear expansion coefficient in to unknown echo path, multichannel structure is obtained by converting the cascade model. For multichannel state-space model, recursive Bayesian was presented in the form of adaptive kalman algorithm in discrete fourier transform (DFT). Two variants of stable implementation were finally described that is fully diagonal multichannel state space frequency domain adaptive filter (FD-MCSSFDAF) and sub-matrix diagonal multichannel state-space frequency domain adaptive filter (SD-MCSSFDAF). Neda Ehtiati and Benoît Champagne [27] in February 2013, introduced a new concept of echo canceller in mixed domain for discrete multi-tone (DMT) based system. This was obtained by providing a general

decomposition of toeplitz matrix at transmitter. Based on this general concept, they proposed new mixed domain canceller (MDC). In which the LMS adaptive filter weight update was done in transform domain. Linear constrained mixed-domain cancellers (CMDC) with additional constraints was also presented to improve performance.

There are certain problems with transform domain normalized least mean square (TDNLMS) and other variants of LMS algorithm. One of them, is their sensitivity level to the excitation signal varies accordingly over time as in case of some signals like: audio signals and speech. Mean square error increases as the excitation signal become low and also the transformation coefficients become small. Another problem with the TDNLMS is there sensitivity to the modeling errors. This problem mainly occurs in active noise control (ANC). To overcome all these problems a regularization term is added in this algorithm. This concept was proposed by S. C. Chan, Y. J. Chu and Z. G. Zhang [28] in April 2013. Therefore, conventional TDNLMS algorithm is now named as regularized transform domain normalized LMS (R-TDNLMS) algorithm. Regularization enhances the convergence speed and at low excitation it reduces the estimation variance. This concept give rise to the proposed concept, variable regularized TDNLMS (VR-TDNLMS) algorithm. Convergence rate of VR-TDNLMS is faster and steady state excess MSE (EMSE) is lower than conventional TDNLMS algorithm. Regularized FxLMS (R-FxLMS) is a filtered based algorithm.

TABLE 1: Tabular review on transform domain adaptive filters

S No.	Algorithms	Findings	Applications	Citation
1	UD factorization of fast RLS algorithm	It is numerical stable, its mean and mean square variance are smaller than SFTF algorithm and other square-root RLS algorithms.	Analyzing Noise and speech signals, real time implementation of ADPCM-based wideband audio coder and in UD Kalman filter	[8]
2.	SO_TRLMS	It improves the convergence speed of TRLMS.	It handles narrow-band Gaussian interference and multi-tone jammers	[9]
3.	DCTLMS	Fast converg-ence rate than standard LMS	It is used in inverse modeling of channel equalization and system identification	[10]
4.	PU-DCTLMS	Reduction in Computational complexity, It provide good convergence performance for both real signal and first-order Markov	It is used in all real time applications	[10]
5.	FBLMS / fast block LMS	It reduces the computational complexity and has Fast convergence rate	It is used in all real time application	[11]
6.	Unconst-rained PFBLMS	It has slow convergence rate	It is used in various acoustic applications like echo cancellation, for broadband spatial filtering	[11]
7.	Schedule-constrained PFBLMS algorithm	It is considered as the best alternate in implement-ation of the PFBLMS algorithm as it do not have a problem of high Computational complexity nor does it suffer from slow converg-ence rate mode.	It is used in all real time application	[11]
8.	TDVSS	Speed of convergence significantly increases in comparison to conventional TDLMS	It is used in system identification and many other real time application	[13]

S No.	Algorithms	Findings	Applications	Citation
9.	NMCFLMS	Good efficiency and fast converg-ence	It is used for three channel acoustic system with long impulse response to generate an accurate channel and also for speech processing for time delay estimation	[14]
10.	PBFDRAP	It reduces the algorithmic complexity as, it is the fast version of previous algorithm	It is used in acoustic echo cancellation setup, it provides better system estimate than conventional PFBLSMS	[15]
11.	Alias-free SADF	This algorithm is first applied to pradhan's. pradhan's SADF is always stable and its EMSE is smaller when compared with full-band adaptive filter	It is used in many practical applications	[16]
12.	Low-complexity transform-domain (TD) adaptive algorithm	Fast converg-ence	Acoustic echo cancellation (AEC)	[17]
13.	HBNLMS	Computational complexity is reduced in comparison to FBNLMS	It is used to implement various video and audio estimation problems adequately. It is inherently used in VLSI design.	[18]
14.	Fast Newton transversal filter (FNNTF) algorithm	It is more flexible to implement	It is used in different practical application	[12]
15.	Fast block-exact LMS (FELMS)	Computational complexity is reduced , numerically stable and when long adaptive filters are required, it is considered as a good alternative of block-exact FNNTF algorithm	Acoustic echo cancellation	[19]
16.	FWTDLMS-PU	It work proficiently even if restricted number of sub-filters are appointed for updation at each iteration.	It is used in various real time applications	[20]
17.	FFT based transform-domain FTAF	It has good degree of fault tolerance without using redundant hardware in zero padding	It is used in many real time applications	[23]
18.	STFT	MSE is improved with non-linear components	Non-linear system identification	[24]
19.	Kalman algorithm in DCT	It is effective for echo cancellation in case of double talk and it changes its echo path	AEC in the existence of an unknown memory less non-linearity preceding before the echo path	[26]
20.	R-TDNLMS	Convergence speed increases and at low excitation it reduces the estimation variance	Echo cancellation	[28]
21.	VR-TDNLMS	It is robust to the power-varying inputs to the algorithm, convergence rate is enhanced and also there is an improvement in steady state EMSE	It is used in active noise control (ANC) systems and acoustic system identification	[28]

III. CONCLUSION

A detailed review on transform domain adaptive filters has been studied and presented. It provides better computational speed, fast convolution, enhances convergence performance as compared to time domain algorithms.

REFERENCES

- [1] Kazuo Murano, Shigeyuki Unagami and Fumio Amano, "Echo cancellation and applications," *IEEE Communications Magazine*, pp. 49-55, January 1990.
- [2] Athanasios P. Liavas and Phillip A. Regalia, "Acoustic echo cancellation: Do IIR models offer Better modeling Capabilities than their FIR counterparts," *IEEE Transactions on Signal Processing*, vol. 46, no. 9, pp. 2499-2504, September 1998.
- [3] John J. Shynk, "Adaptive IIR filtering," *IEEE Acoustic, Speech and Signal Processing. Magazine*, pp. 4-21, April 1989.
- [4] Narayan S, Malibu CA, Peterson A.M, and Narasimha M.J, "Transform domain LMS algorithm," *IEEE Transaction on Acoustics, Speech and Signal Processing*, vol. 31, pp. 609-615, January 1983.
- [5] Shynk J.J, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, pp. 15-37, January 1992.
- [6] Clark, G.A., Mitra, S.K. and Parker, S., "Block Implementation of adaptive digital filters," *IEEE Transaction on Acoustics, Speech and Signal Processing*, vol. 29, pp. 744-752, June 1981.
- [7] S. Gazor and B. Farhang-Boroujeny, "Quantization effects in transform-domain normalized LMS algorithm," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 39, no.1, pp. 1-7, January 1992.
- [8] Alberto Carini, and Enzo Mumolo, "A numerically stable fast RLS algorithm for adaptive filtering and prediction based on the UD factorization," *IEEE Transactions on Signal Processing*, vol. 47, no. 8, pp. 2309-2313, August 1999.
- [9] Chong-ni-Li, Guang-ru Hu and Min-jie Liu, "Narrow-band interference excision in spread-spectrum systems using self-orthogonalizing transform-domain adaptive filters," *IEEE Journal*

- on *Selected Areas in Communications*, vol. 18, no. 3, pp. 403-406, March 2000.
- [10] S. Attallah and S. W. Liaw, "Analysis of DCTLMS algorithm with a selective coefficient updating," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 48, no.6, pp. 628-632, June 2001.
- [11] Kheong Sann Chan and Berhouz Farhang-Boroujeny, "Analysis of the partitioned frequency-domain block LMS (PFBLMS) algorithm," *IEEE Transaction on Signal Processing*, vol. 49, no. 9, pp. 1860-1874, September 2001.
- [12] Kazushi Ikeda, Shigemitsu Tanaka and Youhua Wang, "Convergence rate analysis of fast predictor-based least squares algorithm," *IEEE Transaction on Circuit and System-II Analog and Digital Signal Processing*, vol. 49, no.1, pp. 11-15, January 2002.
- [13] Radu Ciprian Bilcu, Pauli Kuosmanen and Karen Egiazarian, "A transform domain LMS adaptive filter with variable step-size," *IEEE Signal Processing Letters*, vol. 9, no.2, pp. 51-53, February 2002.
- [14] Yiteng (Arden) Huang and Jacob Benesty, "A class of frequency-domain adaptive approaches to blind multichannel identification," *IEEE Transactions on Signal Processing*, vol. 51, no. 1, pp. 11-24, January 2003.
- [15] Koen Eneman and Marc Moonen, "Iterated partitioned block frequency-domain adaptive filtering for acoustic echo cancellation," *IEEE Transaction on Speech and Audio Processing*, vol. 11, no.2, pp. 143-158, March 2003.
- [16] Shigeyuki Miyagi and Hideaki Sakai, "Convergence analysis of alias-free subband adaptive filters based on a frequency domain technique," *IEEE Transactions on Signal Processing*, vol. 52, no.1, pp. 79-89, January 2004.
- [17] K. Mayyas and T. Abounasr, "Reduced-complexity transform-domain adaptive algorithm with selective coefficient update," *IEEE Transactions on Circuit and System-II: Express Briefs*, vol. 51, no. 3, pp. 136-142, March 2004.
- [18] Vasanthan Raghavan, K. M. M. Prabhu and Piet C. W. Sommen, "An analysis of real-fourier domain-based adaptive Algorithms implemented with the hartley transform using cosine-sine symmetries," *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 622-629, February 2005.
- [19] Y. Zhou, S. C. Chan and K. L. Ho, "A new bock-exact fast LMS/Newton adaptive filtering algorithm," *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 374-380, January 2006.
- [20] Samir Attallah, "The wavelet transform-domain LMS adaptive filter with partial subband-coefficient updating," *IEEE Transactions on Circuit and System-II: Express Briefs*, vol. 53, no.1, pp. 8-12, January 2006.
- [21] Elen Macedo Lobato, Orlando José Tobias and Rui Seara, "Stochastic modeling of the transform-domain LMS algorithm," *IEEE Transactions on Signal Processing*, vol. 56, no. 5, pp. 1840-1852, May 2008.
- [22] Shengkui Zhao, Zhihong Man, Suiyang Khoo, and Hong Ren Wu, "Stability and convergence analysis of transform-domain LMS adaptive filters with second-order autoregressive process," *IEEE Transactions on Signal Processing*, vol. 57, no. 1, pp. 119-130, January 2009.
- [23] Chandrasekhar Radhakrishnan and William Kenneth Jenkins, "Fault tolerance in transform-domain adaptive filters operating with real-valued signals," *IEEE Transaction on Circuit and Systems-I: Regular Papers*, vol. 57, no.1, pp. 166-178, January 2010.
- [24] Yekutiel Avargel and Israel Cohen, "Adaptive nonlinear system identification in the short-time fourier transform domain" *IEEE Transactions on Signal Processing*, vol. 57, no. 10, pp. 3891-3904, October 2009.
- [25] Sheng Wu, Xiaojun Qiu and Ming Wu, "Stereo acoustic echo cancellation employing frequency-domain preprocessing and adaptive filter," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 19, no. 3, pp. 614-623, March 2011.
- [26] Sarmad Malik and Gerald Enzner, "State-space frequency-domain adaptive filtering for nonlinear acoustic echo cancellation," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 20, no. 7, pp. 2065-2079, September 2012.
- [27] Neda Ehtiati and Benoît Champagne, "A general framework for mixed-domain echo cancellation in discrete multitone systems," *IEEE Transactions on Communications*, vol. 61, no.2, pp. 769-780, February 2013.
- [28] S. C. Chan, Y.J. Chu and Z.G. Zhang, "A new variable regularized transform domain NLMS adaptive filtering algorithm—acoustic applications and performance analysis," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 21, no. 4, pp.868-878, April 2013.
- [29] Symon Haykin, *Adaptive filter theory*, 3rd edition, Prentice-Hall, 1996.