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Wavelet Based Approach to Solve Eigenmodes in A Rectangular Waveguide

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Abstract — **Wavelet based approach for solving the modes of waveguide is by solving the partial differential equation. Any wavelet expansion approach to solving differential equations is a projection method in which the goal is to use the fewest number of expansion coefficients to represent the solution since it leads to efficient numerical computations. Specifically, we describe a method for constructing the wavelet representation and describe a fast, adaptive algorithm for applying these operators to functions expanded in wavelet basis. Here solutions using wavelet depends on initial and boundary data. Using the wavelet approach to partial derivatives gives sparse matrices which is easier in computation.**

Index Terms: **Multiresolution analysis, modes, rectangular waveguide, Wavelets.**

INTRODUCTION

The ultimate goal is to obtain a basis whose elements are both spatially and frequency localized. The vector expansion coefficient in the basis will provide both spatial and frequency information. Wavelet provide such a basis. We can also compute the DFTs quickly via FFT but Fourier basis is not spatially localized. We obtain a basis that can be computed quickly $w \in l^2(Z_n)$ is such that $B = R_k W_{k=0}^{N-1}$ is an orthonormal basis for $l^2(Z_n)$.Then the coefficient of a vector z in terms of B are the inner products$< z, R_k W >$.Instead of looking for one vector W whose full set of translates form an orthonormal basis we look for two vectors u and v such that the set of their translates by even integers forms an orthonormal basis.

I. REPRESENTATION OF FUNCTIONS IN WAVELET BASIS

The projection of function f(x) onto subspace V_J is by $(P_j \text{ f })$(x)= $\sum_{k \epsilon Z} s_k{}^j \, \varphi_{j,k}(x)$ where P_j denotes the projection operator onto subspace V_J.The set of coefficients $s^j{}_{k\epsilon Z}$, which is referred to as averages is computed by

$$s_k{}^j = \int_{-\infty}^{+\infty} f(x)\varphi_{j,k}(x)dx$$

II. REPRESENTATION OF OPERATORS IN WAVELET BASIS

Here two dimensional wavelet bases are computed by tensor product of two one dimensional wavelet basis functions.

$$\psi_{j,j',k,k'}(x,y) = \psi_{j,k}(x)\psi_{j',k'} , \quad (1)$$

Where $j, j', k, k' \in Z$.Wavelet functions is ψ and the scaling function is φ. Space of the dimension N=2^n.The family of functions

$\varphi_{j,k}(x)$=$2^{-\frac{j}{2}}\varphi(2^{-j}x - k)_{k\epsilon Z}$ forms an orthonormal basis of V_j and a family of functions of

$$\psi_{j,k}(x)=2^{-\frac{j}{2}}\psi(2^{-j}x - k)_{k\epsilon Z} \; . \quad (2)$$

forms an orthonormal basis of W_j.The function φ may be expressed as a linear combination of the basis function of V_{-1}

$$\varphi(x) = \sqrt{2}\sum_{k=0}^{L_f-1} h_k \varphi(2x - k) \; . \quad (3)$$

Similarly

$$\psi(x) = \sqrt{2}\sum_{k=0}^{L_f-1} g_k \psi(2x - k) \; . \quad (4)$$

in multiresolution analysis (MRA) of L^2(R) as

$$\ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset V_{-3} \subset \cdots \quad (5)$$

III. Construction Of Multiresolution Analysis And Daubchies Wavelet Design:

Daubchies D6 wavelet on Z is used to construct a wavelet system on R, with compactly supported wavelets [6].Ingrid Daubchies wavelets are very well localized in space rather than in frequency. N is divisible by $2^P$for some positive integers.$R_{2K}v_{k=0}^{M-1} \cup R_{2K}u_{k=0}^{M-1}$ is a first stage wavelet basis for $l(Z)$.

$\hat{u}(n) = \sum_{k=0}^{5} u(k)\, exp^{-2\pi\, ikn/N}$. (6)

$u_1, v_1 \epsilon$ $l^2(Z)$, $u_2, v_2 \epsilon l^2(Z_{N/2})$, $u_p v_p \epsilon l^2(Z_{N/2^p})$ pth stage wavelet basis can be obtained.Defining $\psi_{-j,k} = R_{2^j k} f_j$and $\varphi_{-j,k} = R_{2^j k} g_j$we call it resulting orthonormal system.Daubchies D6 wavelet basis $l^2(Z)$ where 6 refers to number of nonzero components of u and v.

A. *wavelet with compact support and their computation*:

If $u = u(k)_{k\in Z}$ and

$m_0(\xi) = 1/\sqrt{2}\sum_{k\in Z} u(k)\, e^{-ik\xi}$. (7)

Then $\Pi_{j=1}^{+\infty} m_0(\xi/2^j)$ converges to a function $\hat{\varphi} \in l^2(R)$.A wavelet system for $l^2(Z)$ was constructed with generators u and v that had only six nonzero components[6].

$m_0(\xi) = 1/\sqrt{2}\sum_{k=0}^{N} u(k) e^{-ik\xi}$. (7')

For some positive integers N. Suppose $|m_0(\xi)| \leq 1$ for all $\xi \in R$

and $m_0(0) = 1$.

If $\varphi(\xi) = \Pi_{j=1}^{+\infty} m_0(\frac{\xi}{2^j})$. (8)

Then φ has compact support with supp $\varphi \subseteq [0.N]$. u(k)=0 for k<0 and k>N.

Since $v(k) = (-1)^{k-1} u(1-k)$, (9)

$v(k)$is nonzero only when 0≤1-k≤N or –N+1≤k≤1.

ψ⊆ [-N/2 +1/2, N/2 +1/2]

This is the interval of length N.

IV. Two Dimensional Wavelet Analysis For The Modes In A Rectangular Waveguide:

The waveguide cross section is given by [0, a]x[0, b].The modes in terms of Wavelet transform can be expressed as

$$f(x,y) = \sum_{n,k\in E(m,s)\in F} c[n,k,m,s]\psi_{n,k}(x)\,\psi_{n,k}(y). \quad (10)$$

Where

$$\psi_{n,k}(x) = 2^{\frac{n}{2}}\psi(2^n x - k), \psi_{m,s}(y) = 2^{\frac{m}{2}}\psi(2^n y - s) \quad (11)$$

With $\psi(x)$ as the mother wavelet function.We assume that ψ(x) is concentrated over [0,1] .f(x,y) is the solution to the two dimensional modified Helmholtz equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \boldsymbol{\omega^2}\mu\varepsilon + \lambda^2\right) f(x,y) = 0, (x,y) \in [0,a]x[0,b] \quad (12)$$

The number $k = \omega\sqrt{\mu\varepsilon}$ can be regarded as estimate of the estimate of special frequency of variation of the mode over the guide .The problem of is how to choose the index sets E and F appropriately. Let N(1, x),N(2,x) be respectively the minimum and maximum indices of the resolution in the set E and N(1,y),N(2,y) in the set F respectively specify the x range of the resolution levels. For a given n ∈ [N(1, x), N(2, x)],assuming that the translation factor k varies from $k(1,x,n)$ to $k(1,y,n)$ and for a given m ∈ [N(1, y), N(2, y)],assuming that the translation variables varies from $k(1,y,m)$ to $k(2,y,m)$.In other words ,

$$E = (n,k): N(1,x) \leq N(2,x), k(1,x,n) \leq k \leq k(2,x,n). \quad (13)$$

$$F = (m,s): N(1,y) \leq N(2,y), k(1,y,m) \leq k \leq k(2,y,m) \quad (14)$$

We consider the Wavelet $\psi_{n,k}$.It is aconcentrated over the range $[\frac{k}{2^n}, \frac{k+1}{2^n}]$.For a given n,we want k to a range such that this support covers the x span[0,a] of the wavelength, ie $0 \leq k \leq 2^n a$. Likewise for the wavelengths we want for a given m, s to be such that $0 \leq s \leq 2^m b$.

So in order to analyse the modes in a waveguide first step is to design a compactly supported wavelet, which exists between [0, a] and is zero at x=0 and x=a. We generate a one dimensional wavelet first. Meyer and Daubchies wavelets are two such wavelets which are compactly supported. Here Daubchies D6 wavelets have been designed.

V. Simulation And Results:

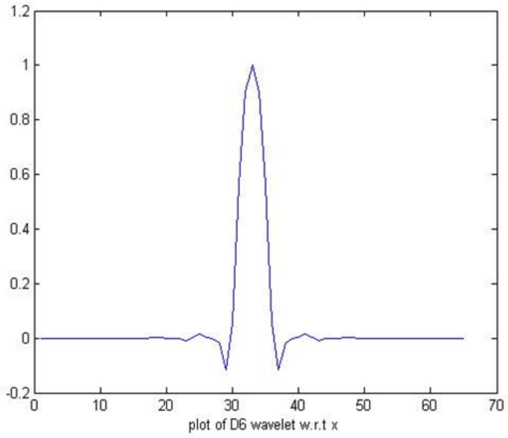

Fig1. plot of φ(x) mother wavelet function w.r.t x using the discretization method

orthogonality of φ(x) and ψ(x) is checked. We take the inner product of φ(x) and ψ(x) over the interval [a, b]

A. Solving partial differential equation with the help of wavelets:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right]\psi(x,y)=0 \qquad (15)$$

where ψ(x, y)

Represents two dimensional wavelets. Plugging in the expansion of f(x, y) into the Helmholtz equation gives

$$\psi(x,y)=\sum_{\substack{N_1\le n\le N_2\\ k_1(n)\le k\le k_2(n)\\ K_1(n')\le K'\le K_2(n')}} c[n,k,n',k']\,\psi_{n,k}(x)\psi_{n',k'}(y)$$

$$\sum \boldsymbol{c[n,k,n',k']}\,(\psi''_{\boldsymbol{n,k}}(x)\psi_{\boldsymbol{n',k'}}(y) + \psi_{\boldsymbol{n,k}}(x)\psi_{\boldsymbol{n',k'}}''(y)) + \boldsymbol{\lambda}\sum \boldsymbol{c[n,k,n',k']}\,\psi_{\boldsymbol{n,k}}(x)\psi_{\boldsymbol{n',k'}}(y)=0 \qquad (16)$$

Multiplying the equation with $\psi_{m,s}(x)\,\psi_{m',s'}(y)$ we get

$$\sum_{\boldsymbol{n,k}} \boldsymbol{c[n,k,n',k']\,\delta_{m'n'}\delta_{s'k'}} < \psi''_{\boldsymbol{n,k}}\psi_{\boldsymbol{ms}} > + \sum_{\boldsymbol{n',k'}} \boldsymbol{c[m,s,n',k']} < \psi_{\boldsymbol{m's'}}\psi''_{\boldsymbol{n'k'}} > + \boldsymbol{h^2 c[m,s,m',s']=0}; \qquad \textbf{(17)}$$

Where $h^2=\omega^2\mu\epsilon+\gamma^2$. (18)

This forms a matrix-

$$Ac+\lambda^2 c=0 \qquad (19)$$

$$det(\boldsymbol{A}+\lambda I)=0 \qquad (20)$$

$$\boldsymbol{\omega^2}\mu\epsilon+\gamma^2=-\lambda(\text{the Eigen value})$$

Modes in a waveguide: are defined by**-**

$$h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 \qquad (21)$$

Where TE_{10} mode is dominating mode. Similarly $TE_{10}, TE_{11}, , TE_{20}$ modes also exist**.**

Results

Simulating equation 17 we get the modes propagated inside the rectangular waveguide.

following modes at different dimensions of waveguides -

at **a=0.5,b=0.25** 3 modes $h^2=-0.9838-0i, \boldsymbol{h^2}=-0.3018-0i$,dominant mode($\boldsymbol{h^2}=-0.6559+0i$) at $f_c=\frac{h}{\sqrt{2\pi*\sqrt{\mu\epsilon}}}$

at **a=0.5,b=0.25** and taking $\boldsymbol{f_c}=1.5*10^8$,$h^2-0.13820i, dominant\ mode\ (h^2=-0.0333-0i,-0.0608+0i,3.16j,3.14j)$

Conclusion

Using wavelet based method Helmholtz equation is resolved. All these simulations are carried out in Mat lab. Applying discrete signal analysis, the work became more simple to analyse the electromagnetic wave problems. This numerical method gives us scope to find propagation of waves in waveguides of different types and shapes. Also there is a wide scope to implement it on DSP chip. Similar methods can also be applied to optical waveguides.

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